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Exercises "Numerical Methods in Fluid Mechanics"
Winter 2023/24 - 1

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1. Give short answers

    1. What is a Tensor?
    2. What is a *continuum*?
    3. Try to explain the *Eulerian Coordinate Framework* and the *Lagrangian Coordinate Framework*.
    4. What is the Reynolds Transport theorem?
    5. What are the fundamental conservation principles leading to the Navier-Stokes equations?
    6. What is an incompressible material? What does it mean in terms of an equation?
    7. Why is the Navier-Stokes equation nonlinear although a "Navier-Stokes material" assumes a linear material law?
    8. What is the meaning of the Reynolds number?
    9. What parts of the Navier-Stokes model are "exact" and where does "modeling" start?
    10. What is a "weak formulation" and why is it called "weak"?
    11. What is a Galerkin discretization?
    12. What is the fundamental principle in constructing finite element spaces? What are the typical steps?
    13. What is "Galerkin orthogonality"?
    14. What does "best approximation property" mean?
    15. Who do linear finite elements look like on a triangular mesh and how do the quadratic fintie element basis functions look like on a 1d mesh (intervals)?
    16. What is the "inf-sup condition"? And what follows from it?

2. An infinite long river

$$\Omega = \{(x, y) \in \mathbb{R}^2 \,:\, 0 < y < 1\}$$

is in motion. The deformation field $\hat{\mathbf{u}}(\hat{\mathbf{x}}, t)$ for a particle $\hat{\mathbf{x}} = (\hat{x}, \hat{y})$ is given as

$$\hat{\mathbf{u}}(\hat{\mathbf{x}}, t) = \begin{pmatrix} \hat{y}(1-\hat{y}) \\ 0 \end{pmatrix} t.$$

a) Show that the velocity in Lagrangian coordinates and Eulerian coordinates is given by

$$\hat{\mathbf{v}}(\hat{\mathbf{x}}, t) = \begin{pmatrix} \hat{y}(1-\hat{y}) \\ 0 \end{pmatrix}, \quad \mathbf{v}(\mathbf{x}, t) = \begin{pmatrix} y(1-y) \\ 0 \end{pmatrix}.$$

Why is there no difference between both viewpoints here?

b) Assume that two ducks are floating in the river with initial position $\hat{\mathbf{x}} = \mathbf{x}_1(0) = (0, 0.2)$ and $\hat{\mathbf{x}}_2 = \mathbf{x}_2(0) = (0, 0.2 + \delta)$ for a small $\delta > 0$. Compute the Deformation gradient as seen from duck $\mathbf{x}_1$. What does it say about the relation of the two ducks?

c) Compute the strain rate tensor, again for duck $\mathbf{x}_1$. What does it say about the relation of the two ducks?

3. Proof a the Reynolds Transport Theorem in 1d: *Let $V(t) = (a(t), b(t)) \subset \mathbb{R}$ be a one-dimensional material volume moving with velocity $v(x,t)$. We observe an interval $(a(t), b(t))$. Show that*

$$\frac{d}{dt} \int_{a(t)}^{b(t)} \varphi(x,t)\, dx = \int_{a(t)}^{b(t)} \frac{d}{dt}\varphi(x,t) + v(x,t)\frac{d}{dx}\varphi(x,t)\, dx$$