Institut für Analysis und Numerik
Universität Magdeburg
Thomas Richter thomas.richter@ovgu.de

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Aufgabe 2.1: –

We consider the networks

1. $1 \mapsto 2n \to 1$
2. $1 \mapsto \underbrace{2 \to 2 \to \cdots \to 2}_{n} \to 1$

with $2n$ neurons each with ReLU activation (no activation for the output). Both networks depict piecewise linear functions. Give an estimate of the maximum number of kinks depending on n for both networks.

Aufgabe 2.2: –

We approximate the function

$$f(x,y) = \exp\Big(1 + 2\sin\big(2x - y + 3y^2 - 2xy + 1\big)\Big)$$

on $[-1,1]^2$. Try to approximate the function using standard l^2-loss

$$l = \sum_{i=1}^{N} \|f(x_i,y_i) - \mathcal{N}(x_i,y_i)\|^2$$

using $N = M^2$ uniform training points in $[-1,1]^2$. Check the approximation usign $(2M)^2$ uniform testing points.
Try the following network architechtures (number of inputs, neurons in hidden layers, ..., output dimension)

1. $2 \mapsto 2 \to 1$
2. $2 \mapsto 4 \to 1$
3. $2 \mapsto 8 \to 1$
4. $2 \mapsto 4 \to 4 \to 1$
5. $2 \mapsto 2 \to 2 \to 2 \to 1$

Use ReLU activation in all but the output layer. Try $M = 100$.

a) Visualize the results $\mathcal{N}(x,y)$ and the error $|f(x,y) - \mathcal{N}(x,y)|$ on $[-1,1]^2$ for all networks.

b) Count the number of kinks or the number of polygons in the trained networks.

Hint: not so easy. You can try with a 1d-example such as $f(x) = \exp(\sin(6x))$ on $[-1,1]$. Also, ReLU networks are not training easily. You can use a sigmoid activation function and analyse the error - as in a) - for small networks with same number of neurons and different depth.